TEACHING NOTE 18-04:
PRICING AND VALUING LIBOR INTEREST RATE SWAPS WITH OIS DISCOUNTING

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Prior to the financial crisis of 2008-2009, interest rate swap pricing was fairly simple. With LIBOR being the benchmark rate, one could easily value, say, a pay-fixed swap as a set of bonds, one issued at a fixed rate with the proceeds used to purchase a bond that pays a floating rate. Although interest rate swaps do not involve payment of the notional, the par values of the long and short bonds offset to produce the same cash flows as the swap.[1] In such a situation, the floating-rate bond with par value of 1 will also have a market value of 1 at each settlement date. As such, the present value of the floating payments plus the hypothetical notional of 1 will equal 1 at the issue date, which considerably simplified pricing. The swap fixed rate will then be the coupon rate on a fixed rate bond that forces the bond to be valued at its par value of 1.

The financial crisis brought attention to the fact that LIBOR might not necessarily be the appropriate rate at which to discount the cash flows on a swap. In the period after the crisis, the Dodd-Frank Act and actions taken by market participants, resulted in the collateralization of most swaps, largely through the use of clearinghouses. Thus, most vanilla interest rate swaps are essentially default-free. LIBOR, however, is a rate that reflects the potential for a London bank to default. Thus, LIBOR has a default premium built into it, meaning that it is higher than it would be if the banks were considered default-free.

In 2007 LIBOR was also tainted by a scandal in which London banks apparently conspired to manipulate the rate they quoted to the British Bankers Association. These quotes rates were used to compile the official daily quoted LIBOR, which sets the rates on many loans and the payoffs of many derivatives. There were many reviews and calls to replace LIBOR, but to date the only changes were that LIBOR began to be handled by the Intercontinental Exchange and has much more oversight. As such, LIBOR has survived as a benchmark rate, and continues to be used in floating-rate loans and derivatives. In fact, it is not well-understood by scholars, practitioners, and regulators that LIBOR is used in derivatives not because of an arbitrary

[1]This result is true provided the bond interest payment dates are aligned with the swap settlement dates, and the floating-rate bond pays interest at LIBOR by setting the rate at the beginning of the settlement period and paying the interest at the end of the settlement period. In addition, the day count adjustment applied to the annual interest rate on the long and short bonds must follow the same convention as on the swaps.

preference for it, but because the derivatives largely exist to hedge the underlying loans, which continue to be pegged to LIBOR.

Thus, some financial economists see an inconsistency in the fact that LIBOR is used to set payments on derivatives and also to discount those payments. The discounting is clearly penalizing the derivative for a likelihood of default that for the most part is non-existent.[2]

As current thinking has evolved, the industry has come to believe that a more appropriate rate for risk-free cash flows would be the Fed Funds rate. Fed funds are non-collateralized short-term loans, usually overnight, in which banks borrow and lend the excess reserves they have at the Federal Reserve. The Fed's wire transfer system facilitates these loans. Hence, many years ago they came to be referred to as Fed Funds, a term that is somewhat misleading in that they are not actually Federal or Federal Reserve money. The Fed funds rate is also sometimes referred to as the OIS rate, whereby OIS stands for Overnight Indexed Swap. An OIS is an actual swap in which the rate is based on the accumulated overnight Fed funds rate. As such, discounting at the Fed funds rate is also sometimes called OIS discounting.

This new twist causes a kink in the pricing of swaps. No longer would one be able to assume that the floating leg of the swap could be replicated with a LIBOR-based floating-rate bond, as that bond would be discounted at LIBOR and return to a value of 1 at each settlement date, which renders the pricing rather easy. Let us review how pricing works under a pure LIBOR-based model and show why pricing cannot work the same way with OIS discounting.

LIBOR Discounting

Let us consider a floating-rate bond in which the final payment with the principal repayment is at time T. At that date, the bond pays LIBOR that was set at time T-1 plus the principal.[3] Letting L_{T-1} be LIBOR at the T-1 date that is paid at the T date. Adding the principal, the total payment at T is $1 + L_{T-1}$. If we step back to time T to value that note, we would discount that amount by dividing by the factor $1 + L_{T-1}$, because that rate is the current one-period rate. Hence, we would obtain a value of the floating-rate bond 1 of T-1. We can then repeat the procedure by stepping back to T-2, T-3, etc. until we reach the issue date. We would then find that the value of the floating-rate bond on each reset date and at the issue date would be 1.[4] So

[2]It is also easy to see the effect on swap pricing. One approach to swap pricing is to take the difference in the forward rates and the fixed rate and discount this difference to a value of zero. If the discount rate used is too high, the fixed rate on the swap will be too low.

[3]For illustrative purposes, we shall ignore the day-count adjustment.

[4]Of course, the value would not be 1 between the reset dates but determining the value is quite simple. Let us say we are between reset date T-9 and T-10. The rate was set at T-10 and the interest based on that rate will be paid at the next date, T-9. So we simply discount that interest plus 1, 1 being the value of all subsequent payments, over the remaining period from where we are between T-9 and T-10 and T-9. This calculation gives the value of the floating payments plus hypothetical notional of 1. The value of the fixed

when we are discounting at the same rate to which the coupons are pegged, pricing and valuation are simple.

For review, let us look at an example with LIBOR discounting. Table 1 shows the basic information.

Table 1. Swap Pricing with LIBOR Discounting Example

LIBOR Term Structure					
Days	**LIBOR Rate**	**DF**	**LIBOR Swap Rate**		**Fwd Rate**
180	0.0265	0.986923	0.028776		0.031088
360	0.0290	0.971817			

To verify the swap rate of 2.8776%, we use the standard formula.

$$\left(\frac{1-0.971817}{0.986923+0.971817}\right)\left(\frac{360}{180}\right)=0.028776$$

Of course, this result can also be obtained by discounting the forward rate. First, note that the forward rate is 3.1088%,

$$\left(\frac{0.986923}{0.971817}-1\right)\left(\frac{360}{180}\right)=0.031088$$

Using the 180-day spot rate as the first payment and using this floating rate as the second payment, we obtain the present value of the floating payments as

$$(0.0265)\left(\frac{180}{360}\right)0.986923+(0.031088)\left(\frac{180}{360}\right)(0.971817)=0.028183$$

Also, note that this value is the numerator in the calculation above for the fixed rate, i.e., 0.028183 = 1 – 0.971817. The present value of the fixed payments at unknown rate *R* is

$$R\left(\frac{180}{360}\right)(0.986923+0.971817)$$

Solving for *R* gives

$$R\left(\frac{180}{360}\right)(0.986923+0.971817)=0.028183$$
$$R=0.028776$$

Hence, this result verifies using forward rates that the swap fixed rate is 2.8776%.

payments plus hypothetical notional of 1 is easily found, because those payments are known. The difference in these two value is the value of the swap.

OIS Discounting

Now, with OIS discounting, let $O_{T\text{-}1}$ be the OIS rate at time T-1 applicable to cash flows to be received at time T. Therefore, the value at time T-1 of the final payment at time T would be $(1 + L_{T\text{-}1})/(1 + O_{T\text{-}1})$, which is obviously not 1. Since the value of the floating-rate component does not return to a value of 1 on the reset dates nor at the issue date, it is not possible to value a LIBOR-based swap discounted at the OIS rate as though it were the issuance of one bond with the funds used to purchase the other. It might appear that we could do so by simply discounting the LIBOR payments at the OIS rate, but the problem is that both the LIBOR payments and the OIS rates are unknown beyond the current period. We do not have the benefit of assuming a value of 1 of the floating-rate bond on each reset date and at the issue date.

Now let us return to time T-1 for the LIBOR swap to be discounted at the OIS rate. Consider, however, a different swap that is based strictly on the OIS rate. That is, the floating payments are determined by the OIS rate and, clearly, those payments will be discounted at the OIS rate. At time T, the OIS floating-rate note will pay $1 + O_{T\text{-}1}$. Stepping back to time T-1, we discount this amount to obtain a time T-1 value of $(1 + O_{T\text{-}1})/(1 + O_{T\text{-}1}) = 1$. Obviously this procedure is simple, except that this payment at time T and its value time T-1 do not replicate the value of the floating leg of the LIBOR swap with OIS discounting, because that leg pays the rate $L_{T\text{-}1}$.

We can, however, use information from a basis swap to pull off a trick.[5] A basis swap is a swap in which both legs pay a floating rate, but the floating rates are different. In other words, there is no fixed leg on a basis swap. There are simply two floating legs pegged to different rates.

Consider a basis swap that is initiated at time 0 and expires at time T. When it expires, it will pay $O_{T\text{-}1}$ and receive $L_{T\text{-}1} – s_{T\text{-}1}$, where $s_{T\text{-}1}$ is the difference between LIBOR and the OIS rate. The spread is deducted from the LIBOR receipt because LIBOR is more than OIS. The combined payment at time T for the floating-rate note plus the basis swap is $1 + O_{T\text{-}1} – O_{T\text{-}1} + L_{T\text{-}1} – s_{T\text{-}1} = 1 + L_{T\text{-}1} – s_{T\text{-}1}$. Since the spread is essentially default-free, we can value it by discounting at the OIS rate. Thus, the value of the floating side would be $(1 + L_{T\text{-}1} – s_{T\text{-}1})/(1 + O_{T\text{-}1}) = (1 + L_{T\text{-}1})/(1 + O_{T\text{-}1}) – s_{T\text{-}1}/(1 + O_{T\text{-}1})$. Of course, we do not know what the spread will be, but there is a way to side-step that problem.

What we are trying to do is value a swap pegged to LIBOR with discounting at the OIS rate by using the principal of issuing one bond and using the proceeds to buy another. Let us say

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[5]This technique is similar to what is suggested in Thomas S. Coleman, "Swap Valuation with Dual Curves – Approximations." Working paper. March 10, 2017, and Donald J. Smith, "Valuing Interest Rate Swaps Using Overnight Indexed Swap (OIS) Discounting," *The Journal of Derivatives* 20 (Summer, 2013), 49-59.

we issue a fixed-rate bond discounted at OIS and use the proceeds to buy a floating-rate bond discounted at LIBOR but we also agree to pay the difference in the LIBOR and OIS payments. Such a transaction will replicate a swap pegged to LIBOR discounted at OIS, as we show in the example to follow.

Example of Pricing a LIBOR/OIS Swap

Table 2 shows the OIS term structure.

Table 2. OIS Term Structure

OIS Term Structure		
Days	**OIS Rate**	**DF**
180	0.0225	0.988875
360	0.0255	0.975134

First, let us price the swap using the difference in forward rates approach. The present value of the LIBOR floating rates using OIS discounting is:

$$0.0265\left(\frac{180}{360}\right)0.988875 + 0.030188\left(\frac{180}{360}\right)0.975134 = 0.028260$$

We set this value plus 1 equal to the present value of the fixed rate payments at *R*, as below,

$$0.028260 = R\left(\frac{180}{360}\right)(0.988875 + 0.975134)$$

Solving for *R* gives

$$R = 0.028778$$

While this value is close to the one obtained using LIBOR discounting, 2.8776%, there is a difference, however small in this case.

Now, let us obtain the same solution by using the bonds approach. Suppose we issue a floating rate bond pegged to the OIS rate but add a basis swap in which we pay the OIS rate and receive LIBOR. The floating rate bond will be valued at 1, as the coupon is set to the OIS rate and discounting is done at the OIS rate. The present value of the difference between the LIBOR floating payments and the OIS floating payments can be found, but first we need the OIS forward rate

$$\left(\frac{1 + 0.0255\left(\frac{360}{360}\right)}{1 + 0.0225\left(\frac{180}{360}\right)} - 1\right)\left(\frac{360}{180}\right) = 0.028183$$

Now, we calculate the present value difference in the LIBOR and OIS floating rates, using OIS discounting.

$$(0.0265-0.0225)\left(\frac{180}{360}\right)0.988875+(0.030188-0.031088)\left(\frac{180}{360}\right)0.975134=0.003394$$

Then, the present value of the floating rate OIS bond plus the value of the basis swap is 1 + 0.003394.[6] We set this value equal to the present value of the OIS fixed-rate bond:

$$1.003394=R\left(\frac{180}{360}\right)(0.988875+0.975134)+(1)0.975134$$

Solving for *R* gives

$$R=0.028778$$

And this is the same value obtained from discounting the difference in the forward rates.

So we see that we can price a standard swap pegged to LIBOR with OIS discounting as the issuance of a fixed-rate OIS bond with the proceeds used to buy a floating rate OIS bond adjusted by the present value difference of a LIBOR-OIS basis swap. Nonetheless, to price a floating-rate bond pegged to LIBOR and discounted at the OIS rate, we do need to use the floating rate approach. That is, we use the forward rate as the implied future payment.

[6]Note that the value of the basis swap would be 0.003394. If an actual basis swap were being done, the party paying the lower rate, OIS, would have to add a spread that has a present value of 0.003394.