TEACHING NOTE 13-01:
PRICING AND VALUATION OF ADJUSTABLE INTEREST RATE SWAPS

Version date: November 23, 2015 C:\Users\Don\Documents\Work Folders\Class Material\Teaching Notes\TN13-01.docx

This Teaching Note on the pricing and valuation of adjustable interest rate swaps complements TN97-06, Pricing and Valuation of Interest Rate and Currency Swaps. You may also wish to review TN12-01, Pricing and Valuation of Amortizing Interest Rate Swaps, and TN05-01, The Pricing and Interest Sensitivity of Floating-Rate Securities.

This Note covers a case we call "adjustable" interest rate swaps. These swaps are very common but in reality, they have no special name. They are swaps in which the floating rate adjusts more frequently than the payments are made. For example, one might enter into a swap in which payments are made every six months. In the standard case, the rate is reset on each payment date, interest accrues for six months, and then the interest is paid, at which time a new floating rate is set. In the adjustable variant of this type of swap, the floating rate would be reset multiple times during the six month period. Thus, it might be reset each month at the monthly rate. In other words, interest accrues at a rate that changes periodically until the end of the settlement period, at which time the interest is paid. In the standard case, interest accrues at a rate that is set at the beginning of the settlement period and remains fixed throughout the period.

These swaps exist primarily because the loans they are designed to hedge are structured in this manner. These loans are themselves floating rate securities. In TN05-01 we cover this variation of the standard floating-rate loan. We showed that pricing this instrument is quite simple. Recall that if the discount rate and coupon rate come from the same term structure, meaning there are no credit issues, the standard floating-rate loan returns to its par value at each payment/reset date. In the adjustable floating-rate loan, the payment and reset dates are not the same, there being more of the latter than the former. We showed in TN05-01 that the floating-rate security returns to its par value on the payment dates, but not on the reset dates. Nonetheless, on the reset dates the loan returns to a value that is easy to determine: it is simply the value of the principal plus whatever interest has accrued since the last payment date.

Imagine a \$1 one-year floating-rate loan that resets each month and has quarterly payments. For the first month, the rate is set to 3%. At the end of that month, the balance owed, but not yet payable, is 1 + 0.03(30/360) (using the LIBOR convention). Let us assume the rate is reset to 3.2%. At the end of the second month, the balance on the balance on the loan has accrued to (1 + 0.03(30/360))(1 + 0.032(30/360)). Now let us say the rate is reset to 3.5%. The balance on the loan will accrue to (1 + 0.03(30/360))(1 + 0.032(30/360))(1 + 0.035(30/360)), which equals 1.008105. So the interest owed will be 0.008105 per \$1 borrowed. When the interest is paid, the balance of the loan goes back to \$1. While we know that the loan value will go back to \$1 at the payment date, we are interested in what the loan value will be on the reset dates and in between the reset dates. At any date, we always know what the loan balance will be on the next reset date. It will be the previous balance times the interest factor. Thus, at any time in the first 30 days, we know that the loan balance after 30 days will be (1 + 0.03(30/360)). Hence, we can discount that balance back to the present to get the value of the loan. And on any reset date, we know precisely what the loan balance is.

A swap set up in this manner is very easy to value. It is quite convenient that we require no more information than we would on a standard swap, and in fact, this type of swap will have the same fixed rate as a standard swap. Let us adapt the examples we have previously used in TNs 97-06 and 12-01. Those examples used a term structure of three annual rates. The data are:

r(0,0.5) = 0.0785

r(0,1) = 0.08, r(0,2) = 0.09, r(0,3) = 0.10

B(0,1) = 1/[1 + 0.08(360/360)] = 0.9259

B(0,2) = 1/[1 + 0.09(720/360)] = 0.8475

B(0,3) = 1/[1 + 0.10(1080/360)] = 0.7692

Thus, d_1 = 360, d_2 = 720, d_3 = 1080. For valuation purposes, let the notional be \$10,000,000. A standard swap with payments on those three dates would have a rate of

$$\left(\frac{1-0.7692}{0.9259+0.8475+0.7692}\right)\left(\frac{360}{360}\right)=0.0908$$

Consider an adjustable swap with its rate reset every 180 days. It will have the same fixed rate, but let us make sure we know why. Recall that the above equation for pricing a swap comes from finding the coupon rate on a fixed rate bond that forces its value to par, the notional on the swap. The notional value is the value of the floating rate security that replicates the floating side of the swap. For an adjustable swap, the floating rate security would need to be one that adjusts on the same schedule, but for certain, its value is its par or notional on the initiation date.

Valuation at various points in the life of the swap will, however, be different for the two swaps. Let us consider the swap value on day 90, which is prior to the first reset date, day 180, which is the first reset date, and on day 270, which is after the first reset date and before the first payment date.

On day 90, assume we have a term structure as follows:

r(0.25,0.5) = 0.082, r(0.25, 0.75) = 0.084, r(0.25,1) = 0.085

r(0.25,2) = 0.088, r(0.25,3) = 0.892

B(0.25,0.5) = 1/(1+ 0.082(90/360)) = 0.9799

B(0.25,0.75) = 1/(1+ 0.084(180/360)) = 0.9597

B(0.25,1) = 1/(1 + 0.085(270/360)) = 0.9401

B(0.25,2) = 1/(1 + 0.088(630/360)) = 0.8666

B(0.25,3) = 1/(1 +0.0892(990/360)) = 0.8030

The first 180-day rate was quoted above as 7.85%. So at this point, what we know is that the floating rate part will be worth \$10,000,000(1 + 0.0785(180/360)) = \$10,392,500 on the reset day, day 180, which is 90 days later. So we can discount \$10,392,500 over 90 days to get (\$10,392,500)(0.9799) = \$10,183,611. The fixed payments are worth

$$\begin{aligned}&\$10,000,000(0.0908(0.9401)+0.0908(0.8666)+(1+0.0908)(0.8030))\\&=\$10,399,608\end{aligned}$$

The value of the adjustable swap to pay fixed and receive floating will be

$$\$10,183,611 - \$10,399,608 = -\$215,997$$

The standard swap value will differ. The 360-day rate set at the start will be 8%. Thus, on the next payment date, the floating value will be \$10,800,000. We discount that value back 270 days to get \$10,800,000(0.9401) = \$10,153,080. The value of the pay fixed, receive floating standard swap is

$$\$10,153,080 - \$10,399,608 = -\$246,528.$$

On the reset date, day 180, the floating part of the adjustable rate swap will be worth $10,392,500, a number we calculated above. The floating part of the standard swap will be worth the present value of $10,800,000 discounted back to day 180 at whatever the 180-day rate is on day 180. Any difference in value of the two swaps will come from the difference in these calculations, because the fixed part will be worth the same.

Moving past the reset date to day 270, we would need to know what the 180-day rate was on day 180. Let us assume it is 8.4%. So, we know that the adjustable swap will be worth $10,000,000(1 + 0.0785(180/360))(1 + 0.084(180/360)) = $10,828,985 on the payment date. We would discount this value back 90 days to get the value of the floating part. For the standard swap, we know that on the payment date it would be worth $10,000,000(1 + 0.08(360/360)) = $10,800,000. We would discount this value back 90 days to get the value of the floating side of the standard swap. The fixed value would be the same for both types of swaps, so any difference would arise from differences in the floating sides.

It should be apparent that resetting the rates more often than the frequency of payments is beneficial to the holder of the pay fixed, receive floating adjustable rate swap over the holder of the pay fixed, receive floating standard swap when rates are increasing. In the adjustable swap, the floating payments are going up, while in the standard swap the floating payment is locked in for a longer period of time.