MBA Teaching Note 18-01
The Rule of 72

The rule of 72 is a very handy one in finance. In reality, the rule is really the Rule of 69, but you will see why it is often expressed as the Rule of 72. The rule is a handy mnemonic that enables one to determine how long it would take for an investment to double in value, given a certain compound rate of return.

The problem is expressed as follows. Let T by the number of years and r be the compound rate of return, expressed in decimal form. For example, the investment might earn a compound rate of return of $r = 0.06$ for $T = 10$ years. For every dollar invested, you would end up with

$$\$1(1+0.06)^{10} = \$1.79$$

As you can see, the investment has not doubled in value. It is easy, however, to solve for the value T that would turn \$1 into \$2. Before doing that, however, let us write the problem in general terms:

$$\$1(1+r)^{10} = \$2$$

Now, we solve for T:

$$\begin{aligned} \$1(1+r)^T &= \$2 \\ (1+r)^T &= \frac{\$2}{\$1} \\ &= 2 \\ T\ln(1+r) &= \ln 2 \\ T &= \frac{\ln 2}{\ln(1+r)} \end{aligned}$$

The natural log of 2 is 0.6931. Now, let us look at $\ln(1 + r)$. It so happens that for most reasonable rates of return, the natural log of $1 + r$ is approximately equal to r. Note in the table below.

Rate (r)	**1 + r**	**ln(1+r)**
0.02	1.02	0.0198
0.04	1.04	0.0392
0.06	1.06	0.0583
0.08	1.08	0.0770
0.10	1.10	0.0953
0.12	1.12	0.1133

This approximation gets far less accurate the higher is the rate, but rates higher than those in the table are not particularly realistic over more than a few years. Thus, without too much loss of accuracy, we can approximate $\ln(1 + r)$ as r. The solution is now found as

$$T = \frac{0.6931}{r}$$

Sometimes for convenience, we express the compound rate without the decimals, such as expressing 0.06 as simply 6, letting the rate in this form as R, we have

$$T = \frac{69.31}{R}$$

It has become common, however, to change 69.31 to 72, simply because it is much easier to divide in one's head into 72 than 69.31. Thus, we often have

$$T = \frac{72}{R}$$

The following table shows the results using 69.31, 72, and the precise result, which is obtain by using ln(1 + *r*) and not approximating it with *R*.

	Rate	**Rule of 69**	**Rule of 72**	**Exact**	
	0.01	69.31	72.00	69.66	
	0.02	34.66	36.00	35.00	
	0.03	23.10	24.00	23.45	
	0.04	17.33	18.00	17.67	
	0.05	13.86	14.40	14.21	
	0.06	11.55	12.00	11.90	
	0.07	9.90	10.29	10.24	
	0.08	8.66	9.00	9.01	
	0.09	7.70	8.00	8.04	
	0.10	6.93	7.20	7.27	

As you can see, the Rule of 72 works reasonably well, and thus, most people use it.