**MBA Teaching Note 17-01**
**The Effect of Costs on Investment Performance**

In practice, it is not possible to invest money without incurring some costs. The principal costs are commissions and bid-ask spreads, as well as management fees and expenses.

Commissions are fees paid to brokers who arrange transactions. A broker connects a buyer to a seller. For this service, a broker charges a fee. This fee can be a fixed fee for a transaction or it can be based on the amount of money traded or the number of shares or bonds. It can also be a combination, such as a fixed minimum plus a percentage of the monetary amount of the transaction. Commissions represent the cost of accessing the market.

Bid-ask spreads are implicit costs paid in dealer markets. A dealer market is a market in which an entity called the dealer stands ready to buy or sell. If an investor wants to buy, the dealer is willing to sell. If the investor wants to sell, the dealer is willing to buy. The dealer quotes a price at which he is willing to buy, called the bid price or bid, and a price at which he is willing to sell, called the ask price or ask. The ask is higher than the bid. Let us say an investor wants to trade a stock that is quoted at 45 bid, 45.50 ask. If the investor wants to buy, the dealer is asking 45.50. If the investor wants to sell, the dealer is asking 45.5. Typically we would assume that the dealer believes that 45.25 is the fair price, whereupon the dealer marks the ask price up to 45.50 and the bid price down to 45.00.

The bid-ask spread is the ask price minus the bid price and represents coverage of the dealer's cost and a profit margin to compensate him for the time spent and risks taken. In this case, the bid-ask spread is 0.50. The bid-ask spread is a cost in that if the investor bought, at 46, and then immediately sold, at 45.50, before there is any change in the price, the investor would incur a loss of 0.50. This cost is considered the cost of liquidity or immediacy, which represents the price the investor pays for the privilege of immediately buying or selling. Assuming no change in the bid-ask spread, for the investor to profit, the price must move by at least the spread.

For example, if the fair price is 45.25, with a bid of 45 and an ask of 45.50, let us say the investor buys at 45.50 and then immediately decides that the transaction was a mistake. He then immediately sells at 45.00. He loses 0.50. In order to make money, the fair price must move up from 45.25 to 45.75. At a fair price of 45.75, the ask would be 45.50 and the bid would be 46. So, the fair price has to reach at least 45.75, a move of the bid-ask spread of 0.50.

When an investor hires a professional manager to manage his money, there is also a cost. There are basically two types of investment managers. You can hire a private manager, an individual who engages with you on a personal basis to manager your money. This person will change a management fee. Alternatively, you can invest in a mutual fund or hedge fund. The fund will charge an expense ratio, which will include the management fee and any other costs of managing or administering the service. These fees will also cover commissions, but they do not cover bid-ask spreads, which are embedded into the prices at which transactions occur.

The fees for professional managers are usually expressed as a percentage. Let us look at the math of how this fee affects performance. Let $R_{t+1}$ be the rate of return earned on the investment over the period $t$ to $t+1$. Then $R_{t+1}$ is the rate over the period $t+1$ to $t+2$, etc. So, if you invested \$1 from period $t$ to period $t+n$, your returns would be $R_{t+1}, R_{t+2}, R_{t+3}, \ldots, R_{t+n}$. In the impossible situation of there being no costs, your \$1 would grow to:

$$\$1(1+R_{t+1})(1+R_{t+2})(1+R_{t+3})...(1+R_{t+n})$$

Now, let us assume that at the end of the year, the manager takes out a portion of your money, $c$, as the fee. For example, if the manager charges 1.5%, then $c = 0.015$. Then your investment would grow to

$$\$1(1+R_{t+1})(1-c)(1+R_{t+2})(1-c)(1+R_{t+3})(1-c)...(1+R_{t+n})(1-c)$$

This can be written as

$$\$1(1+R_{t+1})(1+R_{t+2})(1+R_{t+3})...(1+R_{t+n})(1-c)^n$$

In the above problem, there are different returns from year to year. You can express these returns as single compound rate, such that

$$\sqrt{(1+R_{t+1})(1+R_{t+1})(1+R_{t+3})...(1+R_{t+n})}-1=\bar{G}$$

We are using $\bar{G}$ because the compound rate is the geometric mean. Then the final value of the portfolio is

$$(1+\bar{G})^n(1-c)^n$$

The effect of this cost can be quite tremendous. Let us work a couple of problems.

**Problem 1**:
Suppose you invest $1,000 for 30 years, earning a compound rate of 8.5%. If the annual cost is 0.7%, how much money would you have after 30 years.

$$\$1,000(1+.085)^{30}(1-.007)^{30}=W$$
$$W=\$9,362.02$$

**Problem 2**
Suppose you invest $25,000 for 40 years at a compound rate of 8.25%. After 40 years you end up with $375,000. What is your annual cost?

$$\$25,000(1.0825)^{40}(1-c)^{40}=\$375,000$$
$$c=0.0115$$

**Problem 3**
An approximate average cost for a mutual fund invested in stocks is 0.7%. Some index funds and ETFs can be obtained for as low as 0.05%. Suppose you invest $5,000 today at a compound rate of 8.75% for 30 years. Calculate how much money is saved from using the cheaper fund, with your calculation expressed in terms of money 30 years from now.

Answer: We want to know the difference in the money we would have in 30 years with a 0.7% cost versus 0.05%.

$$\$1,500(1+.0875)^{35}(1-.0005)^{35}=\$27,766.01$$
$$\$1,500(1+.0875)^{35}(1-.0070)^{35}=\$22,097.30$$
$$\text{Difference}=\$5,668.71$$

Note the huge difference. The higher cost takes about 20% of your money.