MBA Teaching Note 14-02

Why Present Value Must Be Lower the Further Out, Even if the Discount Rate is Lower

One of the first principles of finance is that money today is worth more than money at a later date. This statement means that the present value of a dollar at a future date has to be worth less than it is today. Most people grasp this concept pretty quickly, because they know that, assuming a non-zero interest rate, their bank account will have more money later than today. They would certainly rather get $1 into their account today than to get it later.

But it is a little known fact, at least among students, that this statement must hold *even if we use a lower rate to discount the future cash flow*. Suppose r_2 is the discount rate on money received in two periods, and r_1 is the discount rate on money received in one period. Differences in these two rates arise from a term structure of interest rates that is not flat. In simple terms, two-year bonds need not offer the same return as one-year bonds.

The present value of a dollar two years from now is thus, $\$1(1 + r_2)^{-2}$ and the present value of a dollar one year from now is, $\$1(1 + r_1)^{-1}$. For simplicity, we can just drop the $1. Now, if present values further out are lower than present values closer in, we are saying that

$$(1+r_2)^{-2} < (1+r_1)^{-1}$$

That is, the present value of $1 in two years at rate r_2 is less than the present value of $1 in one year at rate r_1.

One might imagine, however, that if r_2 is low enough, the discount effect might mean that the present value of $1 in two years at rate r_2 could be more than the present value of $1 in one year at rate r_1. Therefore, let us assume that

$$(1+r_2)^{-2} > (1+r_1)^{-1}$$

As it turns out, there is now an opportunity to earn a profit at no risk. Suppose we borrow $(1 + r_2)^{-2}$ for two periods, promising to pay back $1 two periods later. Since the amount we borrowed exceeds the amount we would receive for lending at one period, we can make a one-period loan of the amount $(1 + r_1)^{-1}$ that promises to pay us $1 in one period. And because of the inequality, we would have a little bit left over.

Now, one period later, the loan we made as a lender comes due. We take its $1 payoff and reinvest it for one period at whatever rate holds at this point. Let's call that rate r^*. In two periods, we will have $(1 + r^*)$. Our two-period loan then comes due and we owe $1. Now, as long as r^* is greater than zero, we will clearly have enough money, because the one-period loan we made paid $1 at time 1 and we rolled it forward and earned a little more interest. In fact, we would still have that money left over that we started with, which would likely have been lent for two periods to earn additional interest at the rate r_2. So if this condition holds, investors would execute this strategy in large numbers, which would force a realignment of rates until the profit opportunity disappeared.

Without getting into details, this example shows a condition in which the forward rate is negative. A forward rate is a rate at which you could agree today to borrow at a future date. Negative forward rates do not exist: no one could get someone to agree to lend at a later date at a negative rate. In short, money to come later is always worth less than money received earlier, regardless of the discount rate.