MBA Teaching Note 10-03
The Fundamental Principle of Arbitrage

One of the most important concepts in finance is the principle of *arbitrage*, which is sometimes called the *Law of One Price* or LOP. In this note, I want to explain at an extremely fundamental level how this principle works.

The LOP states that identical goods cannot sell for different prices. For example, if a good sold for $2 in one market and $3 in another, someone would buy the good for $2 in the cheaper market and sell it for $3 in the more expensive market. The increased demand in the cheaper market would push its price up, and the increased supply in the more expensive market would push its price down. Ultimately, the good must sell for a single price in all markets. These buying and selling transactions are referred to as arbitrage, and the person doing it is an *arbitrager* (sometimes *arbitrageur*, from its French origin). The arbitrager is said to be earning an *arbitrage profit*. "Arbitrage" is also a verb in that one can "arbitrage" in the sense of executing an arbitrage strategy.[1] Other related terms are "arb" (a noun representing the person doing the arbitrage) or "to arb" (a verb representing the activity) and its variants such as "arbs" and "arbbing."

The concept of arbitrage encompasses a tremendous number of ideas that are central to understanding finance and in particular how financial markets operate. For example, a stock could sell on two different exchanges, but it should presumably carry the same price on each exchange. Minor deviations could exist to reflect costs associated with buying a stock on one market and selling it on another. In more advanced treatments of finance, there are derivative securities that can produce precisely the same result as the original primary security. For example, the performance of a stock can be replicated precisely by buying a call option (the right to buy the stock at a fixed price), selling a put option (the right to sell the stock at a fixed price), and buying a risk-free zero-coupon bond. This concept is very basic in the treatment of options but beyond the scope of what I am covering in this note. But it is sufficient to say that combinations of assets that can replicate other combinations of assets should sell for the same price.

These rules do not mean that the pair of Levis that you can buy at Macy's will sell for the same price as at Wal-Mart. Product markets do not operate this efficiently and people pay differently for service and convenience. It would be impractical and too costly to buy something at Wal-Mart and attempt to convince Macy's customers to buy it from you, even if you could offer a cheaper price, one higher than the Wal-Mart price but lower than the Macy's price.[2]

Basic Illustration

To illustrate the basic idea with a very simple example from product markets, let us assume a barter economy. This is an economy in which money does not exist. People produce certain specialized goods and services and trade those with others for their respective goods and services. Consider a farmer's market in a barter economy. There are three farmers, one of whom produces bread, another produces eggs, and another produces milk. We will assume a loaf is one unit of bread, a dozen large eggs is one unit of eggs, and a gallon is one unit of milk.

It will be easy to see the relative prices of these goods. We can choose any of the three goods as the base good and express the prices of the other two relative to it. Let us use bread as the base good. Now we want to know how many units of eggs is equivalent to one unit of bread and how many units of milk is equivalent to one unit of bread. Of course, these questions would

[1]The expression "arbitrage strategy" thus reminds us that "arbitrage" can also be an adjective.

[2]It should probably be noted, however, that much of Wal-Mart's success has been its ability to get customers who might be willing to pay more at Macy's to instead shop at Wal-Mart, either because convenience and service are not worth the difference in price, perhaps because Wal-Mart is quite competitive in terms of convenience and service.

be answered in this barter economy by the process of supply and demand. We can hardly answer them without making some assumptions. I looked up the present prices of these goods and with some rounding, I found that one unit of bread was \$2.50, one unit of eggs was \$2.00, and one unit of milk was \$4.00. So I will use these prices to get us going on the problem.

How many units of eggs is equivalent to one unit of bread? The price of bread, \$2.50, would buy 1.25 units (i.e., 1.25 dozen) of eggs.[3] This is because the price of bread, \$2.50, would buy \$2.50/\$2.00 = 1.25 units of eggs.

How many units of milk is equivalent to one unit of bread? The price of bread, \$2.50, would buy 0.625 units (i. e., 0.625 gallons) of bread. This is because the price of bread, \$2.50, would buy \$2.50/\$4.00 = 0.625 units of milk.

Thus, we can say that the following relationships hold:

$$B = 1.25E = 0.625M,$$

where the symbol B represents one unit of bread, E represents one unit of eggs, and M represents one unit of milk. Thus, 1.25E is 1.25 units of eggs.

Any violations of this rule would result in an arbitrage opportunity. Suppose for example that market conditions change such that the bread farmer now requires 1.5 units of eggs for one unit of bread, and the egg farmer now requires 0.75 units of milk for 1.25 units of eggs. We now have

$$B = 1.5E$$
$$1.25E = 0.75M$$

This forces the price of bread in relationship to milk to change. Someone with one unit of bread could get 1.5 units of eggs and someone with 1.5 units of eggs could get (1.5/1.25)(0.75) = 0.9 units of milk. Thus, the person with one unit of bread could swap it for 1.5 units of eggs, which he could then use to swap for 0.9 units of milk. Thus,

$$B = 1.5E$$
$$1.25E = 0.75M$$
$$B = 0.9M$$

Or, written compactly as[4]

$$B = 1.5E = 0.9M$$

Note that the price of bread in terms of milk is more expensive. The milk farmer must now give up 0.9 units of milk to get one unit of bread. This is as it has to be. Think through the logic.

Bread is more expensive in relation to eggs.
Eggs are more expensive in relation to milk.
Thus, bread must be more expensive in relation to milk.

Of course, the farmers would negotiate these prices but equilibrium would be obtained provided that each farmer not only needs the good he produces but also each of the other two. A price that provides each with what he needs and clears the market would generally occur.

Note also that we can write these relationships as follows:

$$\left(\frac{\text{Bread}}{\text{Eggs}}\right)\left(\frac{\text{Eggs}}{\text{Milk}}\right)=\left(\frac{\text{Bread}}{\text{Milk}}\right)$$

Here Bread/Milk is the number of units of bread per unit of milk, Eggs/Milk is the number of units of eggs per unit of milk and Bread/Milk is the number of units of bread per unit of milk. Using the new prices, we have

[3]We know these commodities are not now sold in fractional units, such as 15 eggs, but in a barter economy, they probably would be.

[4]Note here that we say that 1.5E = 0.9M and we just above said that 1.25E = 0.75M. You should be able to see that these are equivalent prices.

$$\left(\frac{\text{Bread}}{\text{Eggs}}\right)\left(\frac{\text{Eggs}}{\text{Milk}}\right)=\left(\frac{\text{Bread}}{\text{Milk}}\right)$$
$$=\left(1/1.5\right)\left((1/0.75)(1.25)\right)=\left(1/0.9\right)$$
$$=(0.667)(1.667)=1.111$$

Note that in this formulation, we have to specify the prices inversely, (e.g, bread per unit of eggs instead of the other way around). But the left-hand size equals the right-hand side. We could of course play around and invert one or two of these and see what happens to the other. We just have to make sure the math holds. If the math does not hold, there is an arbitrage opportunity, which we cover a little further down.

Note also that the above relationship implies that

$$\left(\frac{\text{Bread}}{\text{Eggs}}\right)\left(\frac{\text{Eggs}}{\text{Milk}}\right)\left(\frac{\text{Milk}}{\text{Bread}}\right)=1$$

The math clearly says this is true. Plug the numbers in to convince yourself.

Now let us see what happens if the price in the market does not conform to this rule. Using the new prices of 1.5 units of eggs for one unit of bread and 1.25 units of eggs for 0.75 units of milk, we found that there should be a price of 0.9 units of milk for one unit of bread. Suppose the milk farmer does not understand this principle and offers one unit of milk for one unit of bread. The bread farmer will gladly take it. Let us assume the bread farmer is a little faster than the egg farmer and spots this arbitrage opportunity.

He takes one unit of bread and exchanges it for one unit of milk. His one unit of milk can be exchanged for (1/0.75)(1.25) = 1.667 units of eggs.[5] So by exchange one unit of bread, he obtained 1.667 units of eggs, which exceeds his quoted price of 1.5 units of eggs. Why should he stop with just one unit of bread? In fact, he would prefer to sell his entire inventory to the milk farmer and then exchange his milk for eggs. The milk farmer will recognize the increased demand and will increase his price, offering less milk for bread. As we noted in the above paragraph, the milk farmer failed to adjust to market conditions and offered too much milk for the bread, but he will catch on quickly.[6]

Of course, if each farmer needs some of his own product and some of the other two products, the bread farmer would keep some of his bread, but he would certainly realize the largest profit by selling all of his bread to the milk farmer and none to the egg farmer. The egg farmer can meet his personal demand for bread by buying from the milk farmer who is probably wondering how he allowed himself to acquire so much bread.

Arbitrage in the Financial Markets

What we have been describing with this barter economy is identical in principle to the global currency market. Instead of bread, eggs, and milk, we could replace them with dollars, euros, and yen. Each currency can be exchanged in relation to each other. If dollars become more expensive in relation to euros and euros become more expensive in relation to yen, the dollar must become more expensive in relation to yen. Currency are even easier to trade and move around than are bread, eggs, and milk, so arbitrage opportunities are quickly exploited and prices must align.[7]

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[5] 0.75 units of milk will get him 1.25 units of eggs, so one unit of milk will get him proportionately more units of eggs.

[6] He will probably catch on when he realizes that the bread farmer wants to trade his entire inventory to him.

[7] The principle of arbitrage can also be used in currency markets to find the exchange rate for two currencies that do not typically trade against each other. For example, suppose there is no market for the exchange of Thai baht (THB) against New Zealand dollars (NZD). The US Dollar (USD) does trade against NZD and THB. Thus, one could exchange THB for USD and USD for NZD. What would be the exchange rate

Ultimately the law of one price must prevail in well-functioning markets. In our barter market, the LOP can be stated in many ways, depending on which commodity you start with, but one example would be that *the price of bread in terms of milk must be the product of the price of bread in terms of eggs and the price of eggs in terms of milk. It can be no other price*. If it is, someone will figure it out and earn an easy (arbitrage) profit.

Going beyond the currency world, relationships between financial instruments leading to the law of one price are very common. For example, it is possible to take a coupon-paying bond and sell bonds that have claims on each coupon and the final principal payment. The sums of the values of these newly-constructed bonds must equal in value the original bond. As noted earlier, certain derivatives such as options (as well as other derivatives such as futures, forwards, and swaps) can produce equivalent results as the original security on which they are based. Hence, they must be priced equally. A portfolio of securities if traded as a single security must be priced at the sums of the prices of the component securities.[8] And in the most obvious case, a single security trading on different financial markets must have the same price in each market.

We mentioned that there are reasons why the law of one price may not hold precisely. There are sometimes costs, perhaps even significant costs, involved in exploiting arbitrage opportunities. There are also sometimes regulations that block certain types of transactions from occurring. But one thing can be counted on: *market participants understand these relationships*. So you must too.

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implied for THB against NZD? (USD/NZD)*(THB/USD) = THB/NZD. That is, the product of USD per NZD and THB per USD would be the number of units of THB one would get for NZD.

[8]Exchange-trade funds or ETFs are securities that trade like individual securities but are comprised of individual securities, held in a trust.