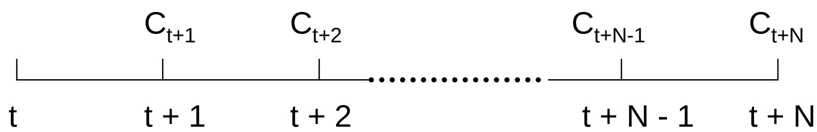
MBA Teaching Note 09-01

Summary of Present Value and Net Present Value Concepts

The objective of present value/net present value problems is to find the value at one point in time, t, of a stream of cash flows occurring over a future period of time. Consider the figure below:

Figure 1. Series of Unequal Payments



This diagram suggests that we might want to find the value at t of the N cash flows C_{t+1}, C_{t+2}, …, C_{t+N}.

> Extremely Important Point: *t could represent today, time 0, or some other point in time in the future.* The symbol t is just a generate point in time. More on this later.

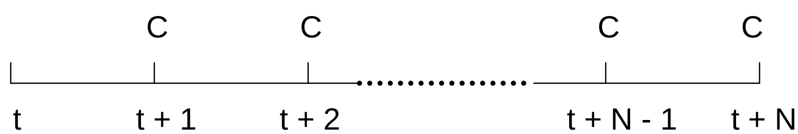
The present value at t is denoted as PV_t and is found in the following manner.

$$PV_t = \frac{C_{t+1}}{(1+r)} + \frac{C_{t+2}}{(1+r)^2} + ... + \frac{C_{t+N}}{(1+r)^N}.$$

Note how each cash flow is discounted over the number of periods from t until that cash flow occurs.

A special case we sometimes encounter is when the cash flows are all equal, i.e., $C_{t+1} = C_{t+2} = … = C_{t+N} = C$, we call this an annuity. See Figure 2.

Figure 2. Annuity



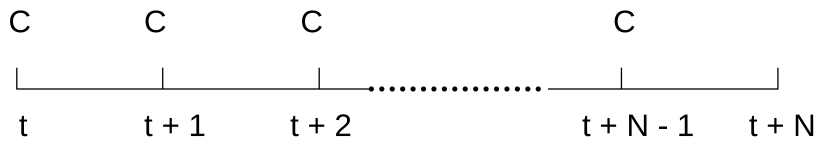
Although you could work this problem by discounting each individual cash flow as above, there is a simpler method and it should be used to save time. Noting that the first payment occurs one period later, we call this stream of cash flows an ordinary annuity and find its present value at t in the following manner:

$$PV_t = C\left(\frac{1-(1+r)^{-N}}{r}\right).$$

> Extremely Important Point: *The assumption that the first payment occurs one period later than time t is absolutely critical.*

Suppose the problem changed slightly such that the first payment occurs today, but there are still N payments, as follows in Figure 3.

Figure 3. Annuity Due



Note that with N payments, the last one occurs at t + N – 1. With the first payment starting at t, this is called an annuity due. Think of this as an annuity of N-1 payments, plus a lump sum value at the start at t. Therefore, we should find the present value at t of an ordinary annuity of N – 1 payments and add the present value of the first payment, C, which of course, has a present value of C:

$$PV_t = C + C\left(\frac{1-(1+r)^{-(N-1)}}{r}\right).$$

(As noted in class, there are a couple of other ways of getting the answer.)

In many cases, we are calculating the present value as of today, time 0. Thus, the subscript t would be 0. But streams of cash flows could start later. Suppose today is time 0 and the first of a series of cash flows does not start until time 10. What is the present value, that is, the value at time 0? Suppose we determine PV_t (whether an ordinary annuity or annuity due), and this would be PV_9. Then $PV_0 = PV_9/(1 + r)^9$.

> Extremely Important Point: *If you calculate a present value as of some future date, t > 0, and fail to see why you must still bring this back to the present to get PV_0, you are missing a major concept.*

Now consider the same problems illustrated in the figures above, but with compounding, i.e., finding the future value. Go back to Figure 1 and find the future value at time t + N of these payments. The answer is obtained as follows:

$$FV_N = C_{t+1}(1+r)^{N-1} + C_{t+2}(1+r)^{N-2} + \ldots + C_{t+N}.$$

> Extremely Important Point: *Make sure you see why the first payment compounds for N-1 periods, the second for N-2, and the last does not compound at all. Look at the figure again and think about how the interest accrues.*

Now consider Figure 2 and find the future value of an ordinary annuity. This is found as

$$FV_N = C\left(\frac{(1+r)^N - 1}{r}\right).$$

Good question: What is the present value at t of FV_N? Is it $PV_t = FV_N/(1 + r)^N$? Or is it, PV_t,

$$PV_t = C\left(\frac{1-(1+r)^{-N}}{r}\right),$$

the amount we obtained above for an N-period annuity? Create a problem and work it both ways. If you do not get the same answer, you are making a mechanical mistake.

> Extremely Important Point: *If you do not see why these two answers should be the same, you are missing a concept. That is even worse than a mechanical*

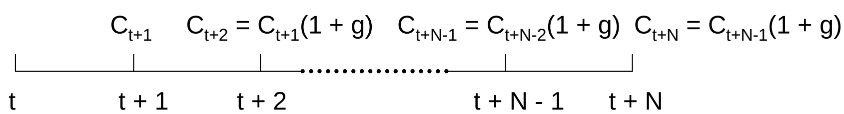
mistake. The point is: the present value of the future value of an annuity is the same as the present value of the annuity. Likewise, the future value of the present value of the annuity is just the future value of the annuity.

Now consider Figure 3, the annuity due. With the last cash flow occurring at t + N – 1, we would typically want to find the future value as of t + N – 1. You should be able to see that this is an ordinary annuity of N – 1 cash flows, plus the initial cash flow compounded for N – 1 periods:

$$FV_{t+N-1} = C\left(\frac{(1+r)^{N-1}-1}{r}\right) + C(1+r)^{N-1}.$$

In some cases we want to analyze growing streams of cash flows. Start with C_1 given and assume that all remaining cash flows are successively higher by the factor g, a growth rate. See Figure 4.

Figure 4. Growing Stream of Cash Flows



Note that every cash flow is higher than the one before it by the factor 1 + g. For some cases, such as stocks, it is reasonable to assume that these cash flows, which are dividends, go on forever. In that case, if the growth rate is not more than the discount rate, the present value is as follows:

$$PV_t = \frac{C_{t+1}}{r-g}.$$

Again, *the stream of cash flows might not start at time 1. Time t might be some future date t. What do we do if we want the value today?* Easy. $PV_0 = PV_t/(1 + r)^t$.

Also note that as long as you know the growth rate and any cash flow at any point in time, *you can easily obtain any cash flow at any later time by simply compounding.* (Example: Suppose a seer tells you that your salary in 10 years will be $300,000 and that you will get a 5% raise in the 11^{th} year. Can you determine your salary for year 11?)

Another important thing to remember is that the term *present* is usually used before the term *value* to get *present value*. Calling it present value is such common practice that we can hardly afford not to do it. But it adds absolutely nothing to put *present* in front of *value* and even creates unnecessary confusion. We are interested in value at some arbitrary time t. If t = 0, we are getting the value today. If the cash flows start at some future date, such as five years from now, the above formulas give us $PV_t = PV_5$. Is PV_5 truly the *present* value? Is it the value today, that is, PV_0? If we need the value today, we have to convert PV_5 to PV_0 by $PV_0 = PV_5/(1 + r)_5$. Failure to see this point means you are missing a major concept.

These points are the key ones that capture the *concepts*, not the mechanics of finding value, whether it be present value, net present value or future value. All we are doing is converting cash at one point in time to its value at another point in time.

Final bit of advice: Attempting to work problems like this without drawing a time line on which you place the cash flows is an almost surefire guarantee of failure.